

Problem posing in the proof process identifying creative thinking in mathematics

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Abstract. Creative thinking is an important part in learning mathematics. However, some Mathematics Education students have low creative thinking skills, especially in the proof process in the Real Analysis course. Use the Problem Posing approach to determine the quality of students' creative thinking abilities in the Real Analysis course. Therefore this study aims to describe the potential of students as designers, in creative thinking in the proof of mathematics. This research is a qualitative research category, with a perspective-constructive approach. A total of 61 sixth semester students take Real analysis courses in the 2019/2020 Academic Year as research subjects. The student studied in two heterogeneous mathematics education classes, from one urban tertiary institution participating in this study. By using descriptive statistics and Pearson correlation can be obtained this research information. The results obtained by the problem posing condition can find a greater difference than the equivalent conditions, about two-thirds of students are able to make in some cases the original equivalent, as well as the relationship between student achievement on Sequences and Series material with originality found at the middle level. This type of research has the potential for lecturers and students to assess the level of student understanding of certain mathematical topics, concepts, or procedures.

1. Introduction

Creative thinking is very necessary in the world of education, especially in the field of mathematics. Creative thinking must be grown from basic education to college. Because creative thinking has a relationship with achievement, [1]. According to Goldin [2,3] creative thinking in mathematics must be applied in the world of education, based on the past human being, mathematicians and intelligent people, which greatly contributes to humanity enjoyed by many people.

Every person who studies mathematics craves creative thinking. However, most mathematics education students do not yet have satisfactory mathematical creative thinking skills. Yet according to Sriraman [4] mathematical creative thinking is very important as a whole role in the growth of mathematics. Likewise, according to Kattou [5,6] between creative thinking has a positive relationship with mathematical ability.

Thinking is the biggest activity in mathematics. Original creative thinking is the result of long period of mathematical activity and reflection based on the use of deep and flexible content knowledge [7]. This conception is more consistent with the definition that characterizes creativity as a product or process, rather than subjective experience, because it emphasizes prolonged involvement with the problem situation and reflection on the process used and the product made. It also shows that teaching can influence creativity.



Creativity is expressed as a relative phenomenon in schools [8]. In other words, students' ideas will be considered creative based on their contribution to mathematical knowledge in class or peer groups. These ideas contribute to what Hershkowitz [8]. calls for a "socio-cognitive" approach to examine creative reasoning in the course of class argumentations. Innovative ideas illustrate examples of how to function to resolve previous contradictions in mathematical discussion, and how individual creative ideas function as turning points for the whole class. Central to their analysis are the interesting ideas about the "knowledge agent" and the "follower" who were last influenced by or responded to the ideas expressed by the first.

Creative mathematical thinking for its size and direction cannot be generally defined. In general creativity according to [9] suggests that student creativity can be linked to problem solving performance and can be considered as a combination of Originality, fluency, and flexibility is expressed in solving problems. According to the [10] model, Originality / novelty as searching for unique ways of thinking and unique products of mental or artistic activity. These criteria relate to students' ability to try novels or unusual approaches to a problem. Flexibility associated with changing ideas, approaching problems in various ways and producing various solutions. It illustrates the ability of students to vary approaches or suggest different methods for a problem or situation, and Elaboration as relating to the ability to describe, illuminate and generalize ideas. Such criteria must assess the person's capacity to redefine the problem or situation to create another person. [10] proposes measuring the level of creativity of a particular solution method by calculating the product's originality measure and its flexibility measure.

Mathematical creativity at the school level [11], such as (1) processes that produce unusual (novel) solutions and / or insights.) for a given problem or analog and / or problem, (2) the formulation of new questions and / or possibilities that allow old problems to be considered from a new perspective.

According to Moore [12], for students to experience the pleasure to foster creativity, students must appreciate the activities that will be carried out. If students engage in assignments routinely, students can complete them with satisfaction with satisfying results, so too according Tabach [13] if students engage in tasks that are not routine and complete satisfactorily, it can produce original and useful work so also for students who excel can produce originality from a variety of variations.

A lecturer teaches class slowly by compensating for the condition of students indirectly always fostering students' creative thinking abilities. Because creative thinking is not only needed in the classroom outside the classroom, it is also needed, and this skill is also needed for the future [14]. Thus fostering mathematical creative thinking impacting students can influence the virtue in proving [15].

Problem-posing and problem solving are closely related [16]. That is the interaction of formulating, trying to solve, reformulate and finally solve, problems seen as creative activities. The characteristics of creativity that can be detected in this framework are new things in the formulation of problems or solutions, shifts in direction during the problem-solving process and the number of different solution paths or different solutions. Problem posing refers to producing something new or expressing something new from a collection of data, therefore involving creativity.

A student must be encouraged to explore concepts, create new ideas, and evaluate those efforts to finally make valid evidence, during the process of proof. The idea reinforces that during the process of solving or proving problems, (such as finding examples that are relevant or representing the same concept in various ways) help develop creativity by laying the foundation for creativity in conditions of originality. The settlement process is a student's thinking process in the use of open problems that provides opportunities for students to solve problems to be creative in the conditions of Originality [17]. As for Shriki argues that the ability to generate or obtain actual proof or discovery of new theorems is also a product of creative thinking [18].

Between definitions / theorems, between representations, and between examples must be considered in the learning of deductive thinking. Students must master the definition, theorem, or examples of questions that have been studied previously or studied in other material. According to Karakok [19] students must have a diffuse thought to see the relationship between other theorems and theorems used. Using the definition / theorem, as well as examples of previous problems in the process of proving it at the initial, development, and advanced levels.

The process of deductive thinking at an early level, students must use the definition / theorem that is relevant to the problem in their assignments. Then at the level of development of the relevant theorem the proving process can run according to the problem of the task at hand. The level of progress of students can implement the definition / theorem in proving other tasks. These levels indicate part of the ability to think creatively. If students master these levels, then the process of embedded evidence is more advanced, the action leads to the potential for mathematical creative thinking, [20].

Most Mathematics Education students have difficulty learning deductive mathematics in the Real Analysis course. Because the process of proof in the course Real analysis is a high-level mathematical category. To overcome these difficulties students often practice the questions and are given assignments [21]. Likewise, the problem of proof in the course of Real Analysis courses students experience difficulties, especially in the material Sequences and series. The material therein is to prove convergent sequence and sequence. To prove the convergence sequence can use a definition and several theorems, so students must be observant which theorem is right to prove the problem being worked on [22]. Even this material to prove the convergence in a particular problem can use more than one answer, which means it involves creative thinking. According to Guilford, [23] linking creative thinking with divergent (or pro-production) thinking, thinking or different answers to specific problems.

Savic Encouraging research the ability of mathematical creativity in universities in the process of proof, the nature of the research Savic suggests that lecturers in teaching use creativity in progress in the proof process, so as to be able to foster new ideas in students [20].

Based on the description above, with the problem of posing, students are required to practice making questions and solving them correctly on material lines and series. Furthermore, the purpose of this study is to describe the potential of students as designers, in thinking of creative mathematics in the proof process.

2. Method

This research is a qualitative research category with a qualitative approach which is an approach to construct a statement of knowledge [24], qualitative used in this study is the perspective-constructive approach.

2.1 Research

The first task (Take-a-quiz) used in this study was that students had to complete the problem of the description of the material Sequences and Series. Every student item must finish well and correctly. Every student item must finish well and correctly. Students are allowed to finish in a different way from those used by lecturers or those in the book. Provided that the settlement is in accordance with theorems / definitions and examples of relevant problems. The objectives with the first assignment are (1) students are accustomed to quizzes with description questions, (2) evaluating students on row and series material, (3) attracting students' attention to the good distractor nature of the question description.

The second task (Make-a-quiz) (done the following week after the first assignment), requires students to make questions (problem posing) and solve them properly and correctly on the Sequences and Series material. Make five items about Sequences and Series that solve using (1) item no 1 using Product to be the Sequence, (2) item no 2 using Sequence Ratio, (3) item no 3 using Subsequences, (4) item no 4 using Divergent Criteria, and (5) item no 5 using The harmonic series. In each of the five items, students must use the verification process, as well as solve the problem properly. The questions made must be relevant to the theorem used to prove. Of the five items, students were asked to make questions that were different from what had been taught by the lecturer or those in the book. And students are asked to give some correct answers from each item. Students are expected to give examples of correct answers, and students are accustomed to using the methods used by lecturers in the proof process.

Students in working on the second task, each of the five items given involves a verification process that uses three levels as described above. This second task involves a cognitive process namely

(1) The thought process that leads to test items that require correct and more complex identification, (2) Use the relevant theorem to prove the questions students make, (3) The design of each item created requires a deep understanding of the verification process, and(4) Students must make new questions.

2.2 Research Questions

The purpose of this study is to describe ways that enable students who take the Real Analysis course can encourage to apply original and creative thinking in completing the design of material description Sequences and Series. The questions in this study are:

- 1). How does the student's level of originality in completing the given task give the correct answer to the item designed?
- 2). What is the level of student knowledge in identifying the first and second assignments?
- 3). Can you identify the relationship between student knowledge from the first assignment and the student's originality in building example problems?

2.3 Participants

A total of 61 sixth semester students take real analysis courses in the 2019/2020 Academic Year. The student studied in two heterogeneous Mathematics Education classes, from one urban tertiary institution participating in this study. The student's background has taken courses in Calculus, Advanced Calculus, Differential Equations, and Number Theory.

3. Result and Discussions

3.1 Make-a-quiz activities

Allow responses of each item out of five items is 61 participants multiplied by 3 per item responses is 183 responses. Because some students do not give the three responses needed, or repeat the same answer, so the total number of responses is smaller than optimal. Here are some information on the results of the make-a-quiz distribution in table 1.

Table 1 Distribution results from Make-a-quiz

Item	Equivalent Expression (%)	Non Equivalent Expression (%)	Total
Product to be the sequence	53(29)	72(42)	125
sequence ratio	43(24)	76(42)	119
Subsequences	38(21)	66(36)	104
Divergent Criteria	32(17)	64(35)	96
The Harmonic series	42(23)	58(32)	100

For each item analyzed, the number of equivalent expressions provided by students is about one third of the total number of responses. This figure shows that some students chose to provide more than one response per item. This shows that:

- distribution provided to students, based on unequal items,
- The number of no equivalent expression by students because distractors are much bigger than the equivalent expression.

3.2 Originality

Equivalent expressions given by students for three of the five items subsequently were analyzed in terms of their level of originality. Students' Take-a-quiz results are then sorted from the most remote to the largest. After being sorted out from 61 students, it was divided into 3 levels. One third of the best are ranked and said to be high, medium and low achieving students. For Originality peritem scores based on student achievement levels are presented in table 2.

Table 2. Distribution *Originality*

Level	Item				
	Product to be the Sequence (%)	Sequence Ratio (%)	Subsequences (%)	Divergene Criteria (%)	The Harmonic Series (%)
Low	12(12)	26(43)	10(16)	26(43)	14(23)
Medium	21(34)	33(54)	24(33)	31(51)	22(36)
High	9(18)	24 (33)	6(10)	10(16)	5(9)

Based on table 2, about two-thirds of student responses are categorized as moderate or high levels of originality. In other words, around two-thirds of students are able to create in a number of ideas equivalent to the original.

3.3 Take-a-quiz activities

The overall success rate for each of the five items shows the difference between the levels (about 60%) of the first two items and the items (about 75% or more) of the other three items.

3.4 Correlation

Based on the Pearson Correlation, the relationship between student achievement (Take-a-quiz) on the Sequences and Series material with originality was found to be at the middle level of 57%.

The findings show that students can engage in open assignments of non-routine tasks of this type and solve them in a satisfactory way. In Make-a-quiz, students can build different ideas that are greater than equivalent ideas. Research in the field of mathematics education in particular, emphasizes the importance of creative ideas that are mathematically correct to be useful for solutions of mathematical problems [25].

Problem Posing gives influence to identify and assess mathematical creativity [26]. According to [27] building ideas and putting forward other types of tasks is a modification of problems in the additional form of problem posing, offering individuals the opportunity to produce coherent and consistent creative problems. This definition is another type of assignment that offers the possibility to recognize students' mathematical creativity and also help them overcome fixation. Problem Posing is a powerful tool for identifying and assessing mathematical creativity. Problem Posing is an intermediary or modification of the problems faced and the transition of a new generation. The Problem Posing can be used as a measure of mathematical creativity and can build creativity [28].

Problem solving and creative thinking are supporters of open tasks. This form of task, mathematical problem solving and creativity need not be limited to open problems but that tasks with appropriate constraints can serve as problem solving tasks that stimulate creativity as well. Another parallel is the idea of the level of openness of the problem-solving task more or less. The level of potential creativity in the problem-solving task, [29]. Problem Posing correlates with student power related to mathematical content, especially in the context of problem solving. The strength is an individual part to build cognitive power, namely the power of creative thinking, [30]. Problem Solving with Expertise, is very instrumental in the process of solving individual problems and can raise the ability to think creatively [31]. Likewise the verification process is considered as part of the problem solving process [32].

About two-thirds of the participating students are able to display medium or high levels of originality in the construction of equivalent ideas. The originality of responses is the dominant characteristic in most definitions of creativity, and research is used in the field of mathematics education [33]. Leikin argues that mathematical responses must be original, rare and in accordance with mathematical problems [34], Mann added that mathematical processes, procedures, and algorithms can also be very original [33]. Leikin suggest that originality determines creativity in a way

stronger than fluency and flexibility. also suggested that originality is more a unique characteristic of internal creativity [34].

The relationship between student achievement in sequence and series material with originality is found and at the intermediate level. This finding seems to indicate that each of the two variables considered in this study is related to the other two. According to Kattou his research indicate a relationship between mathematical ability and mathematical creativity [5]. A strong mathematical background is related to the ability to think creatively in mathematics. Mathematical creative thinking ability is a sub component of mathematical ability.

4. Conclusions

Evidence that Problem Posing can contribute to a deeper conceptual understanding of the process of proving the material of the Sequence and Series in the Real Analysis course. Participating students worked on the Take-a-Quiz questionnaire about the description, and on the Make-a Quiz design activity showed that students could apply some important higher-level thinking skills related to this material as well. By identifying and differentiating between equivalent and unequal ideas in the Take-a-Quiz questionnaire, using metacognitive skills, and by generating examples of equivalent and unequal ideas in Make-a-Quiz activities, high achieving students have varying degrees of originality, as well as the relationship between student achievement with originality at the secondary level on the material sequence and series. Since in Higher Education there is little research on creative thinking in the proof process, it needs to be developed.

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